**Title of the Unit:** Introduction to control theory

**Unit code:**

**Total time per student:** 18h lecture 12h tutorial Xh lab Xh project

**Credit number: 3** ECTS

**Master’s degree:**  Mecanics  Robotics & Auto. Cont.  Electrical Eng.

**Master program:** E3A : CIMES  Syscom  IPS

AR :  SAR  ISI

MECA :  MS2  MF2A  EE  CompMech  ACOU  EE APP

**Semester:**  S1  S2  S3  S4

**Teaching language:**  French  English

**Involved students:**  Sorbonne Université  Other (to be specified): ENSAM

**Localization:**  PMC campus  Other (to be specified): ENSAM Paris

**Objectives of the unit:** Control theory is at the interface of applied mathematics and engineering. It has been one of the most active interdisciplinary fields of the past years. The aim of control is to actively manipulate a system as to modify its natural evolution in order to reach a specified goal. In fluid dynamics, control theory applications have epic proportions: transition delay, drag reduction for cars and trucks, lift increase during take-off and landing phases of commercial planes, increased mixing capabilities for chemical reactors, or noise reduction in aeroacoustics applications. By improving the performances or the robustness of a given system, tools for control theory not only enable one to design better processes. It is also one of the most important keys for an engineer to reach pollutants and greenhouse gas reductions, or to transition toward a greener industry.

By focusing on linear time-invariant dynamical systems, the aim of this course is for students to familiarize themselves with the fundamental concepts and methodologies of control theory. These concepts include the characterization of a system based on its observability and controllability gramians, the design of a controller (MPC, robust control, PID, or LQR), as well as the estimation problem and the associated Kalman filter. Being part of a fluid mechanics program, a large emphasis will be given to model-order reduction techniques, a key enabler for real-time control in such a field.

**Knowledge and skills mastered at the end of the unit :** By the end of the class, students will known how to characterize the stability, receptivity, controllability and observability properties of a linear dynamical system. They will moreover be able to setup standard control strategies, both open-loop and closed-loop, for low-dimensional systems. Finally, they will also have basic knowledge of standard model-order reduction techniques.

**Detailed content of the unit :** The course will be organized as follows

* **Lesson n°1 :** *General introduction*
  + General overview of the course – prerequisites, projects, evaluation.
  + Overview and illustration of control theory.
* **Lesson n°2 :** *Mathematical tools for control theory*
  + Linear algebra – under- and over-determined systems of equations, eigenvalue and singular value decomposition.
  + Optimization theory – Quadratic programming with and without constraints, optimality conditions.
* **Tutorial n°1 :** *Numerical linear algebra and convex optimization*
  + Introduction to numpy, scipy, and cvxpy.
  + Linear least squares methods with and without constraints.
* **Lesson n°3 :** *Introduction to linear dynamical systems*
  + Stable and unstable systems.
  + Observability and controlability of a linear system.
* **Lesson n°4 :** *Basics of open-loop control*
  + Formulation of open-loop control as an optimization problem.
  + Pros and cons of open-loop control.
* **Tutorial n°2 :** *Basics of open-loop control*
  + Designing an open-loop control law for a simple mechanical system.
  + Formulation and resolution of the control problem using cvxpy.
* **Lesson n°5 :** *On the importance of feedback*
  + PID controllers.
  + Linear Quadratic Regulator – Lyapunov and Riccati equations.
* **Tutorial n°3 :** *Linear optimal control*
  + Application to the inverted pendulum on a cart.
  + Computing the LQR controller using scipy.
  + Performances and robustness of the controller.
* **Lesson n°6 :** *State observers and Kalman filters*
  + How to infer the state x of a system from limited measurements y ?
  + Kalman filters and the Linear Quadratic Estimator (LQE).
* **Tutorial n°4 :** *Linear Quadratic Gaussian control*
  + Application to the inverted pendulum on a cart.
* **Lesson n°7 :** *Going beyond LQR with Model Predictive Control*
  + Formulation and illustrations of model predictive control.
* **Tutorial n°5 :** *Model Predictive Control*
  + Application of model predictive control to a nonlinear dynamical system.
* **Lesson n°8 :** *Curse of dimensionality and the need for reduced-order modelling*
  + Balanced truncation and Balanced Proper Orthogonal Decomposition.
  + System identification.
* **Tutorial n°6 :** *Reduced-order modeling for flow control*
  + Application to the stabilization of the von Kàrmàn vortex street behind a two-dimensional cylinder.
* **Lesson n°9 :** *Introduction to reinforcement learning*
  + Illustration of reinforcement learning.
  + Pros and cons.
  + Perspectives for the future of flow control.

This program is only indicative. It might change depending on the students preferences or difficulties.

**Prerequisites of the unit :** Control theory relies on mathematical tools coming mostly from linear algebra and optimization theory. Although the most important ones will be introduced during Lesson n°2, being comfortable with linear algebra is an absolute prerequisite. This includes: the rank of a matrix, eigenvalue decomposition, singular value decomposition, vector subspace, etc. Likewise, prior knowledge of optimization theory might be beneficial. This includes: minimizing a quadratic function, gradient descent, etc. Finally, projects and homeworks will make use of the Python programming language. It is thus necessary to know the basics of Python, and ideally of numpy and scipy. Part of these concepts can be included during the remediation courses taught during the first week of the semester.

**Evaluation of the unit (informative) :** Evaluation is divided into three parts: (i) a regular 2 hours-long exam at the end covering most of the concepts seen during class, (ii) a grade based on the tutorials/hands-on sessions, as well as a final project. The final grade will be calculated as a weighted sum of these different evaluations (e.g. 40% exam + 40% project + 20% hands-on sessions).

**Bibliography :**

* Athanasios C. Antoulas. Approximation of large-scale dynamical systems. SIAM Advances in Design and Control. 2005.
* Steve Brunton and Nathan Kutz. Data-driven Science and Engineering – Machine learning, dynamical systems, and control. Cambridge University Press. 2019. The pdf of the book is available for free at https//www.databookuw.com

**Teaching sequence (informative) :**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| week | lecture | tutorial | lab | project | test |
| W1 | 2 x 2h |  |  |  |  |
| W2 | 2h | 2h |  |  |  |
| W3 | 2h | 2h |  |  |  |
| W4 | 2h | 2h |  |  |  |
| W5 | 2h | 2h |  |  |  |
| W6 | 2h | 2h |  |  |  |
| W7 | 2h | 2h |  |  |  |
| W8 | 2h |  |  |  |  |
| W9 |  |  |  |  |  |
| W10 |  |  |  |  | 2h |
| W11 |  |  |  |  |  |
| W12 |  |  |  |  |  |
| W13 |  |  |  |  |  |
| W14 |  |  |  |  |  |

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**Author**: Jean-Christophe LOISEAU, Maître de Conférences @ Arts & Métiers.