

## **Turbulence**

Turbulence is a fundamental phenomenon at the crossroads of out-of-equilibrium physics, engineering, and mathematics, with applications ranging from aeronautics to climate dynamics. Despite the ubiquity of turbulent flows in everyday life, proving the existence of solutions to the Navier-Stokes equations remains one of the Millennium Prize Problems identified by the Clay Mathematics Institute. Meanwhile, physicists and engineers have developed powerful theoretical tools to predict and characterize the statistics of turbulent flows, supported by advancing numerical and experimental methods. The first part of the course will emphasize these modern developments, while the second part of the course will be focused on applying these methods to various forms of turbulence encountered in the climate system.

**I. Homogeneous isotropic turbulence:** Richardson energy cascade, spectral description, Kolmogorov theory.

**II. Chaotic dynamics:** sensitivity to initial conditions, Lyapunov exponents. Loss of predictability and application to weather forecasting.

**III. Wall bounded turbulence** and turbulent boundary layers: law of the wall, wall shear stress.

**IV. Turbulent convection** with and without rotation.

**V. Two-dimensional turbulence** and the formation of large-scale structures.

**VI. Ocean two-dimensional turbulence above topography:** statistical mechanics.

**VII. Turbulent transport in oceans and planetary atmospheres:** effective transport and multiscale description.

### **Prerequisites:**

Basic hydrodynamics (Eulerian or Lagrangian description, Navier-Stokes equation)

No prerequisites are expected for climate dynamics.

### **Bibliography:**

U. Frisch, Turbulence: the legacy of A.N. Kolmogorov, Cambridge University Press.

S.B. Pope, Turbulent flows, Cambridge University Press.

R. Salmon, Lectures on Geophysical Fluid Dynamics, Oxford University Press.

**Schedule:** December to February.

**Credits:** 3 ECTS

**Hours:** 32 hours.