Nonlinear waves and pattern formation

This course aims to provide simple methods to handle so-called complex phenomena described by nonlinear partial differential equations. Systems governed by nonlinear equations display multiple solutions with different symmetries. We study the bifurcations i.e. the transitions between these solutions when a parameter of the system is varied. We show that in the vicinity of these bifurcations, the system is governed by universal equations, normal forms, that mostly depend on the broken symmetries at the transition. We emphasize the analogy with phase transitions, but also point out differences such that limit cycles or chaotic behaviors that do not occur at equilibrium. The following problems are considered:
1. Nonlinear waves in dispersive media: a universal equation that describes the motion of a wave packet, the nonlinear Schrödinger equation. Instability of a quasi-monochromatic wave, generation of solitons. Solitons as particles.
2. Pattern-forming instabilities in hydrodynamics. Amplitude equations from symmetry arguments.
3. Analogy with the mean field description of phase transitions: superfluidity, superconductivity, magnetic domains, commensurable-incommensurable transitions.
4. Broken symmetries and neutral modes. Secondary instabilities described by phase dynamics. Topological defects.
5. Subcritical bifurcations and metastable states. Localized structures. Analogy with the liquid-vapor transition. Nucleation and Maxwell construction. Non potential effects.

Prerequisite: Overall, a basic Master 1 level in applied maths, physics or mechanics is required

Bibliography:

1. G. B. Witham, Linear and nonlinear waves, Wiley (New York, 1974)
2. A. C. Newell, Solitons in mathematics and physics, SIAM (1985)
3. Hydrodynamics and nonlinear instabilities, edited by C. Godrèche and P. Manneville, Cambridge University Press (1998)

Timing: The Course is offered in the second part (december-february) of the M2 year.

Credits: 3 ECTS

Hours: 30 hours.